PRE-AP CALCULUS Name:______________________________

A Finely Crafted O'Brien Unit 1 Opportunity Day

__Calculator Section__: You may use a calculator. Show all work and circle your answer. Use your time wisely; you will be able to earn additional credit after the timed portion of the test by completing Supercorrections. When you finish, put away your calculator and you can come up to get the non-calculator part- you may continue to work on both sections without your calculator.

1. Given $f(x) = 2x - 7$ and $g(x) = x^2 + 3$. Find:

a. $g(t-3)$

b. $(f+g)(2)$

2. The distance formula equation $\sqrt{(10-4)^2 + (k-4)^2} = 4$ seems to indicate that the points (10, k) and (4, 4) are 4 units apart. What is wrong with this statement? Justify your answer.

3. Solve for x.

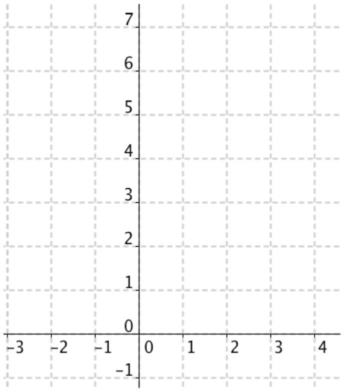
$$\sqrt{x+2} + x = 4$$

4. Let $f(x) = \dfrac{x+4}{x+1}$, $x \neq -1$ and $g(x) = \dfrac{x-2}{x-4}$, $x \neq 4$.

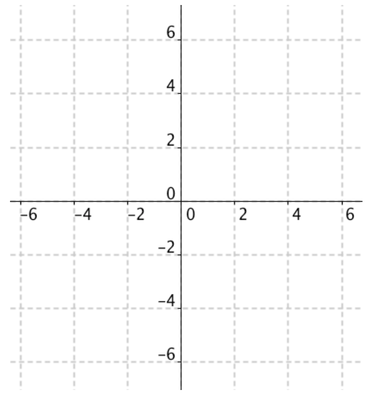
Find the set of values of x such that $f(x) \leq g(x)$.

5. Sketch a graph of the given parametric equations with the parameter t satisfying $0 \le t \le 3$.

$$x = t \text{ and } y = t^3 - 4t^2 + 4t + 2$$



Suppose that the graph at left is only part of the graph of a relation that is symmetric about the y-axis and the origin. Sketch the complete graph of the relation below.



6. The perimeter of a rectangle is 400 meters.

a. If the length of the rectangle is x and the width is y, then write y as a function of x. Use the result to write the area as a function of x.

b. Of all possible rectangles with perimeter of 400 meters, find the length of the shorter side (to the nearest tenth) of the one with an area of 5000 square meters.

7. The functions $f(x)$ and $g(x)$ are given by $f(x) = \sqrt{x-2}$ and $g(x) = x^2 + x$. The function $(f \circ g)(x)$ is defined for all real numbers, except for the interval $a < x < b$. Calculate the value of a and b.

8. Given that $f(3)=4$, evaluate:

a. $f^{-1}(4)$ b. $f(f^{-1}(4))$ c. $f^{-1}(f(3))$

9. Find the equation of the perpendicular bisector of the segment connecting (–2, 3) and (4, 15).

10. Consider the function $f(x)=\dfrac{2x-1}{x+2}$.

a. Find the **domain** of *f*.

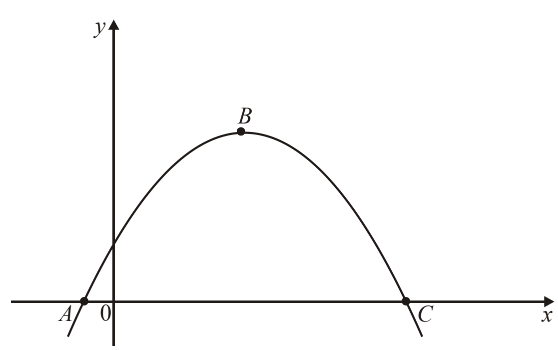
b. Find the **inverse** of *f* and use it to find the **range** of *f*.

inverse: $f^{-1}(x)=$

range of *f*:

11. The diagram shows the parabola $y=(7-x)(1+x)$. The points A and C are the x-intercepts and the point B is the maximum point.

Find the coordinates of A, B and C.

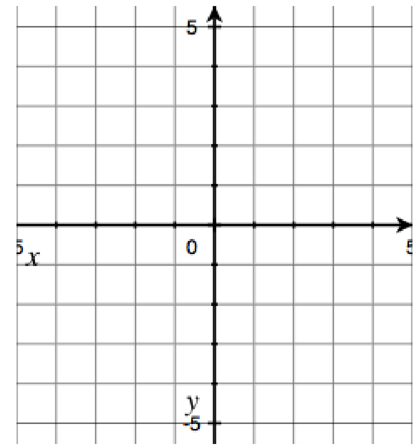


12. Consider the piecewise-defined equation.

$$y = \begin{cases} x^2 & \text{if } x < -1 \\ 1 & \text{if } -1 < x \le 0 \\ [\![x]\!] & \text{if } x \ge 0 \end{cases}$$

a. Sketch the graph of the equation.

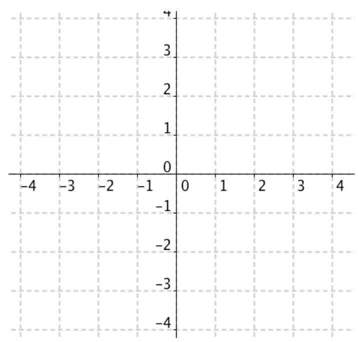
b. Is the equation a function? Why or why not?



13. Duncan is trying to find the inverse of the one-to-one function $g(x) = (x-1)^2 + 2, x \le 1$. His work is at right.

$$y = (x-1)^2 + 2$$
$$y - 2 = (x-1)^2$$
$$\sqrt{y-2} = x - 1$$
$$1 + \sqrt{y-2} = x$$
$$1 + \sqrt{y-2} = g^{-1}(y)$$
$$\therefore g^{-1}(x) = 1 + \sqrt{x-2}$$

a. Use your knowledge of transformations to sketch $g(x)$ and $g^{-1}(x)$ on the same axes.

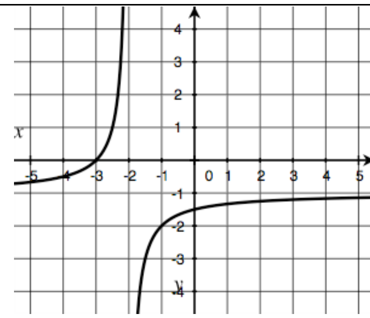


b. How does your sketch show that Duncan made an error in his working?

c. Circle the error, and give the correct inverse for $g(x)$.

f

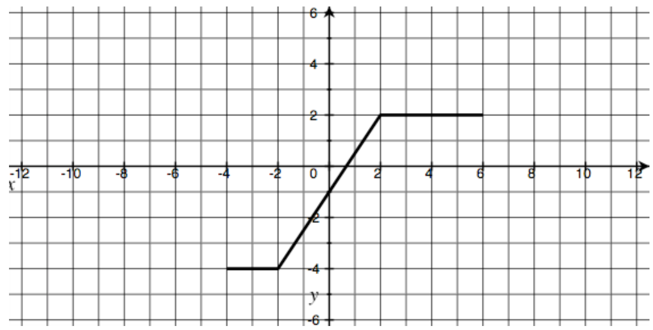
14. The graph of the function $f(x)$ is shown at right.



a. Find $f(-3)$.

b. Find $f^{-1}(-2)$.

15. Consider the interesting function $f(x)$ graphed at right. A second function is defined as $g(x) = f(2x) + 2$.



a. Complete the tables for *f* and for *g*.

x	-2	-1	0	1	2	3
$f(x)$						
$g(x)$						

b. On the same axes, sketch graph of $g(x)$.

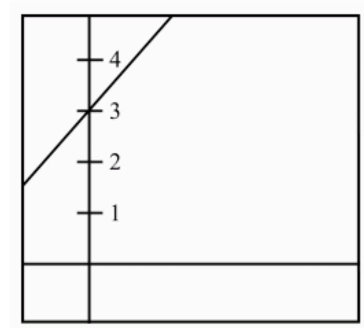
16. Assume the points (–3, 2) and (1, 0) are on the graph of $y = f(x)$. Find *b* so that

a. (–1, *b*) is on the graph of $y = f(x-2)$.

b. (3, *b*) is on the graph of $y = 2 + 2f(x-2)$.

17. Identify two distinct functions *f* and *g* such that $f(g(x)) = g(f(x))$. Explain why your functions satisfy the condition.

18. If you were to zoom out on the graph at right, you would see that it has two x-intercepts. Write the equation of a function that would fit the criteria.



19. Give a well-structured argument to determine whether $f(x) = 2x\sqrt{x^2+3}$ is even, odd, or neither.

Bonus: Solve.

$$\frac{|x|}{x-2} \leq 2$$